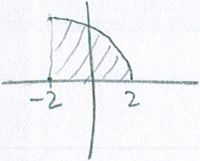
$$\int_{-2}^{2} (5\sqrt{16-(t+2)^2} - t\cos t^3)\,dt$$

$$= \int_{-2}^{2} 5\sqrt{16-(t+2)^2}\,dt - \int_{-2}^{2} t\cos t^3\,dt$$

3

CIRCLE
CENTER $(-2, 0)$
RADIUS 4



$$= 5\cdot\frac{1}{4}\pi 4^2 + 0$$

3

$$= 20\pi$$

2

3

$$(-t)\cos(-t)^3$$
$$= -t\cos(-t^3)$$
$$= -t\cos t^3$$

INTEGRAND IS ODD + CONTINUOUS,

2

SO INTEGRAL $= 0$

2

$$\int \frac{5x-10x^2}{\sin^2(4x^3-3x^2-1)}\,dx$$

3 $u = 4x^3-3x^2-1$

$$\frac{du}{dx} = 12x^2-6x \rightarrow dx = \frac{du}{12x^2-6x}$$

$$= \frac{du}{6(2x^2-x)}$$

$$\frac{5x-10x^2}{\sin^2(4x^3-3x^2-1)}\,dx$$

$$= \frac{-5\cancel{(2x^2-x)}}{\sin^2(4x^3-3x^2-1)} \; \frac{du}{6\cancel{(2x^2-x)}}$$

4 $= -\frac{5}{6}\csc^2 u\,du$ 3

$$\int -\frac{5}{6}\csc^2 u\,du$$

3

$$= \frac{5}{6}\cot u + C$$

3 $= \frac{5}{6}\cot(4x^3-3x^2-1)+C$ 2

$$\int_{-1}^{1} (y^2 - \csc^2 y)\, dy$$

$\csc^2 y$ IS NOT CONTINUOUS

3 @ $y = 0 \in [-1, 1]$

SO FTC 2 DOES NOT APPLY

2

$$\int z^5 \sqrt{1+z^2}\, dz$$

3 $u = 1 + z^2 \rightarrow z^2 = u - 1$

$\frac{du}{dz} = 2z \rightarrow dz = \frac{du}{2z}$

$z^5 \sqrt{1+z^2}\, dz = z^5 \sqrt{1+z^2}\, \frac{du}{2z}$

$= \frac{1}{2} z^4 \sqrt{1+z^2}\, du$

3

3 $= \frac{1}{2} (u-1)^2 \sqrt{u}\, du$ 2

$= \frac{1}{2}(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}})du$

3

$\int \frac{1}{2}(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}})\, du$

3 $= \frac{1}{2}\left(\frac{2}{7}u^{\frac{7}{2}} - \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right) + C$

$= \frac{1}{7}(1+z^2)^{\frac{7}{2}} - \frac{2}{5}(1+z^2)^{\frac{5}{2}}$

3 $+ \frac{1}{3}(1+z^2)^{\frac{3}{2}} + C$ 2

Find $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{ne^{2+\frac{3i}{n}}}$ by finding the corresponding definite integral, and evaluating that integral. **SCORE: _____ / 15 PTS**

$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\Delta x)\Delta x$ WHERE $\Delta x = \frac{b-a}{n}$

$a+i\Delta x = 2+\frac{3i}{n} \longrightarrow a=2$ AND $\Delta x = \frac{3}{n} = \frac{b-a}{n} = \frac{b-2}{n}$

So $b=5$

$f(a+i\Delta x)\Delta x = f\left(2+\frac{3i}{n}\right)\frac{3}{n} = \frac{3}{n}\frac{1}{e^{2+\frac{3i}{n}}}$

SO $f(x) = \frac{1}{e^x} = e^{-x}$

3 EACH

$\int_2^5 e^{-x}\,dx = -e^{-x}\Big|_2^5 = -(e^{-5}-e^{-2}) = e^{-2}-e^{-5}$

Prove that $\frac{\pi}{16} \le \int_{\frac{1}{2}}^{1} x \arcsin x \, dx \le \frac{3\pi}{16}$.

SCORE: _____ / 15 PTS

3 $\frac{\pi}{6} \le \arcsin x \le \frac{\pi}{2}$ FOR $\frac{1}{2} \le x \le 1$

2 EACH EXCEPT AS NOTED

$\frac{\pi}{6}x \le x \arcsin x \le \frac{\pi}{2}x$

$\int_{\frac{1}{2}}^{1} \frac{\pi}{6}x \, dx \le \int_{\frac{1}{2}}^{1} x \arcsin x \, dx \le \int_{\frac{1}{2}}^{1} \frac{\pi}{2}x \, dx$

$\int_{\frac{1}{2}}^{1} \frac{\pi}{6}x \, dx = \frac{\pi}{6}\left(\frac{1}{2}x^2\right)\Big|_{\frac{1}{2}}^{1} = \frac{\pi}{12}\left(1-\frac{1}{4}\right) = \frac{\pi}{\cancel{12}_4} \cdot \frac{\cancel{3}}{4} = \frac{\pi}{16}$

$\int_{\frac{1}{2}}^{1} \frac{\pi}{2}x \, dx = \frac{\pi}{2}\left(\frac{1}{2}x^2\right)\Big|_{\frac{1}{2}}^{1} = \frac{\pi}{4}\left(1-\frac{1}{4}\right) = \frac{\pi}{4} \cdot \frac{3}{4} = \frac{3\pi}{16}$

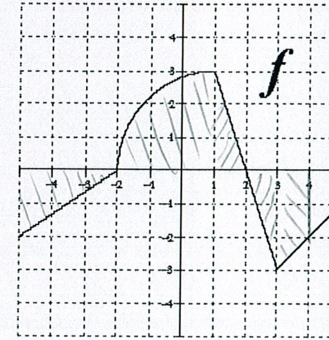
SO $\frac{\pi}{16} \le \int_{\frac{1}{2}}^{1} x \arcsin x \, dx \le \frac{3\pi}{16}$

Using proper English and mathematical notation, state both parts of the Fundamental Theorem of Calculus, as well as the Net Change Theorem. **SCORE: _____ / 15 PTS**

SEE QUIZ 2 SOLUTIONS

Let $g(x) = \int_4^x f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 25 PTS**

NOTE: The graph of f consists of a line, an arc of a circle, and two more lines.



[a] Find $g(-5)$.

② EACH EXCEPT AS NOTED

$$\int_4^{-5} f(t)\,dt = -\int_{-5}^{4} f(t)\,dt$$

$$= -\left[\frac{1}{4}\cdot\pi\cdot 3^2 + \frac{1}{2}\cdot 3\cdot 1 - \left(\frac{1}{2}\cdot 3\cdot 2 + \frac{1}{2}\cdot 1\cdot 3 + \frac{2+3}{2}\cdot 1\right)\right]$$

$$= -\left[\frac{9\pi}{4} + \cancel{\frac{3}{2}} - \left(3 + \cancel{\frac{3}{2}} + \frac{5}{2}\right)\right] = \frac{11}{2} - \frac{9\pi}{4}$$ ③

[b] Find $g'(1)$. **Explain your answer very briefly.**

$g'(1) = f(1) = 3$

[c] Find the $x-$coordinates of all inflection points of g. **Explain your answer very briefly.**

$g' = f$ CHANGES FROM INCREASING TO DECREASING @ $X=1$

DECREASING TO INCREASING @ $X=3$

The table gives the acceleration of a car (in feet per second^2) at various times (in seconds). **SCORE: _____ / 20 PTS**
At time $t = 3$ seconds, the velocity of the car was 21 feet per second.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$a(t)$	4	2	1	3	2	5	4	2	0	-1	-3	0	3	5

[a] Write an expression involving an integral for the velocity of the car at $t = 11$ seconds.

2 EACH EXCEPT AS NOTED

$$v(11) - v(3) = \int_3^{11} v'(t)\,dt = \int_3^{11} a(t)\,dt$$

$$\text{so } v(11) = v(3) + \int_3^{11} a(t)\,dt = 21 + \int_3^{11} a(t)\,dt$$

[b] Estimate the velocity of the car at $t = 11$ seconds using [a], 4 subintervals, and midpoints.

MIDPOINTS
4 6 8 10
3 5 7 9 11

$\Delta t = \frac{11-3}{4} = 2$

$$21 + \int_3^{11} a(t)\,dt$$

$$\approx 21 + (a(4) + a(6) + a(8) + a(10))\Delta t$$

6

$$= 21 \frac{ft}{sec} + (2 + 4 + 0 + -3)\frac{ft}{sec^2}(2\ sec) = (21 + 6)\frac{ft}{sec} = 27 \frac{ft}{sec}$$